Directionality and Crest Length Statistics of Steep Waves in Open Ocean Waters

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ABSTRACT

A new wavelet analysis methodology is applied to open ocean wave height data from the Southern Ocean Waves Experiment (1992) and from a field experiment conducted at Duck, North Carolina, in 1997 with the aim of estimating the directionality and crest lengths of steep waves. The crest directionality statistic shows that most of the steep wave crests are normal to the direction of the mean wind. This is inconsistent with the Fourier wavenumber spectrum that shows a broad bimodal directional spreading at high wavenumbers. The crest length statistics demonstrate that the wave field is dominated by short-crested waves with small crest length/wavelength ratios. The one-dimensional steep wave statistic obtained from the integration of the directional (two dimensional) steep wave statistic is consistent with the one-dimensional steep wave statistic obtained from the one-dimensional analysis at high wave slope thresholds.

1. Introduction

The directionality of ocean surface wave fields is of interest for many reasons. Some measurements of the sea surface topography (Hwang et al. 2000) have revealed a bimodal directional distribution in the two-dimensional (2D) wavenumber spectrum, which is in contrast to the unimodal form proposed by many earlier models. The bimodal directional distribution has, in fact, been shown to yield mean square slope values that are more consistent with field studies than the traditional unimodal directional distributions (Hwang and Wang 2001). In addition, the observed bimodal directional spreading is consistent with the theoretical estimate of nonlinear wave interactions, which moves energy away from the peak frequency and redistributes it into the directions oblique to the mean wind direction (Wang and Hwang 2001). Directional spreading is also important in determining the form of the one-dimensional spectrum (Banner 1990).

There have been very few studies of directional properties of steep and breaking waves. Recently, Melville and Matusov (2002) obtained images of oceanic whitecaps from an aircraft and tracked the evolution of whitecaps using image velocimetry. They were able to calculate the directional breaking wave statistic $\Lambda(\mathbf{c})$ and found a symmetrical angular distribution about the mean downwind direction.

The traditional methods of understanding the directionality of the wave field have been through the use of Fourier-based methods such as the maximum likelihood method (Capon 1969) and the direct two-dimensional Fourier transform. New methods have recently emerged that take into account the wave groupiness of the ocean surface. Donelan et al. (1996) were able to calculate the frequency–directional spectrum by estimating the instantaneous wave propagation directions at various frequencies using a wavelet-based analysis. However, there have been no observational studies of the directionality of steep nonlinear waves. In addition, there has been no study of the statistical nature of crest lengths on the ocean surface.

Recently, direct observations of ocean surface topography were made during the Southern Ocean Waves Experiment (SOWEX) and an experiment conducted off Duck, North Carolina, in September 1997 (hereafter

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termed the DNC experiment). Using these datasets we have developed a data analysis technique to estimate the statistics of nonlinear wave groups based on the wavelet transform. In a companion paper (Scott et al. 2005), we have presented one-dimensional analyses of the steep wave statistics based on the assumption that all steep wave fronts propagate in the mean wind direction. In this paper, we investigate the directionality and crest length statistics of steep wave events in detail by performing two-dimensional analyses of the ocean surface topography.

2. Data analysis

The analysis in this paper was performed on data from the Southern Ocean Waves Experiment and data obtained from a field experiment conducted near Duck, North Carolina, in September 1997. Information on these experiments can be found in Banner et al. (1999) and Hwang et al. (2000). Since the definition of the steep wave statistic based on the wavelet transform is identical to that in the companion paper (Scott et al. 2005), only a brief summary is given below.

a. Definition of the steep wave statistic

The steep wave statistic is defined based on the formulation of the breaking wave statistic $\Lambda(\mathbf{c})$ proposed by Phillips (1985). Using this same idea, the steep wave statistic $\Lambda_T(k, \theta)$ is defined as the total length of steep wave fronts whose wave slope exceeds a set threshold T per unit surface area per unit wavenumber, where an area element in the wavenumber domain is $dk\ k\ d\theta$. Thus,

$$\Lambda_T(k,\theta)dk\ k\ d\theta = \frac{L}{\tilde{A}}, \tag{1}$$

where $\tilde{A}$ is the area of ocean surface, and L is the total length of breaking wave fronts with wavenumbers in the area element $dk\ k\ d\theta$ in the wavenumber domain. The one-dimensional lambda function $\Lambda_T(k)$ is obtained from the integration of $\Lambda_T(k, \theta)$:

$$\Lambda_T(k) = \int \Lambda_T(k, \theta)k\ d\theta. \tag{2}$$

In general, the slope of an individual wave cannot be determined uniquely for random seas with a broad-banded spectrum. Thus the wave slope is defined using the wavelet transform such that the estimated slope of a steep wave event is, in fact, the average wave slope of a small group of waves that are detected by the wavelet transform.

b. Wavelet transform

The wavelet transform of a signal $f(u)$ in this study is defined as

$$Wf(a, s) = \mathrm{Re}\left[\int_{-\infty}^{\infty} f(u)\frac{1}{a^2}\Psi\left(\frac{u-s}{a}\right)du\right] a > 0, \tag{3}$$

where

$$\Psi(a, s) = \Psi(s/a) = e^{-iK_0 s/a}e^{-1/2\,(s/a)^2}, K_0 = 5 \tag{4}$$

is the Morlet wavelet. A signal with a high wave slope event at scale a_0 and position s_0 will have a wavelet transform characterized by a large peak value of Wf at (a_0, s_0). The peak value is proportional to the average wave slope in the neighborhood of s_0.

c. Estimation of the two-dimensional steep wave statistic, $\Lambda_T(k, \theta)$

The observed wave topography data are preprocessed such that the surface elevation is defined at equally spaced grid points both in the along-wind direction and in the cross-wind direction. The wavelet transform is applied to each column vector (in the along-wind direction) of the two-dimensional surface elevation array. The result $Wf(a, s)$ contains an array of high wave slope events. To obtain a distribution of these events, a wave slope threshold is applied over it. All points with a wavelet transform value above a set wave slope threshold T are selected. These points appear as aggregates in local regions of $Wf(a, s)$. From these groups of points, the point of highest value is sought using a nine-point box filter. The local maximum that satisfies the condition of being above the set wave slope threshold is defined as a steep wave crest associated with a wave group. The conversion of the scale a used in the wavelet analysis to the real wavenumber scale k is obtained via a conversion constant, C, such that $k = C/a$. The wavenumber associated with the Morlet wavelet at scale a is taken to be the wavenumber at the peak of its Fourier power spectrum.

5. Conclusions

The wavelet analysis methodology presented here is able to track steep wave events and give estimates of the amount and directionality of high wave slope events that cover a given area of ocean. Analysis of the results shows that high wave slope crests appear over the entire range of wavenumbers resolved and mostly perpendicular to the mean wind direction. Comparison of the steep wave statistic $\Lambda_T(k)$ from the one-dimensional analysis and $\Lambda_{*T}(k)$ obtained by integrating the two-dimensional steep wave statistic shows consistency at moderate wave slope threshold values.

Our crest directionality statistic further confirms that most of the steep wave crests are normal to the direction of the wind even at low wave slope thresholds. This is not consistent with the Fourier wavenumber spectrum that shows a bidirectional spreading at high wavenumbers. The lack of consistency may be partly attributed to the fact that most steep waves are short crested and generate leakage in oblique directions in the Fourier spectrum.